
Language and Logical Pluralism: Some Aspects of a Wittgensteinian Perspective on the Nature of Logic

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Abstract

This essay examines the importance of some aspects of Wittgenstein's post-*Tractatus* work in the realm of discussions on the nature of logic. The first part considers a relationship between certain conceptions of language and certain positions on the nature of logical laws and logical pluralism. Supposing the rejection of mentalism in the field of meaning leads to a rejection of psychologism, it presents some alternatives different from psychologism, based on non mentalistic theories of meaning. One is the Platonistic Fregean approach to language and logic, the other is Carnap's formalist view on both topics. The second part concentrates on Wittgenstein's non mentalistic and non Platonistic proposals about language and his defense of the logical pluralism proposed by psychologists. It compares two periods on Wittgenstein's work after *Tractatus* -the periods of 'calculus conception' and 'language games conception'- and it shows how characteristic notions of Wittgenstein's later conception of language, like 'use', 'language games', and 'forms of life', work on the characterization of logic and specially on the kind of logical pluralism that the author seems to defend in his last period. In doing so, this essay offers an approach to some of the author's considerations about contradictions and the possibility of the existence of a calculus that includes them. This approach emphasizes on the idea of applicability (or use of a linguistic expression) introduced by the author in some of his last works, and in some examples of functional contradictions that can help to understand and complement that idea.

Keywords: Psychologism, mentalism, logical pluralism, use, language games, forms of life.

He [Frege] asks: What should we say if we found people who made judgments contrary to our logical propositions? What should we say if we found people who did not recognize our laws a priori but arrived at them by a lengthy process of induction? Or if we even found people who did not recognize our laws of logic at all and who made logical propositions opposite to ours? He says "I should say 'Here we have a new kind of madness' - whereas the psychological logician could only say 'Here's a new kind of logic' ". This is queer. We wouldn't call a man mad who denied the law of contradiction -or would we?
(Wittgenstein 1939, p. 202)

1 Mentalism, Psychologism and Some Alternatives

Logical psychologism is, in general terms, a philosophical tendency or a theory of logic that establishes a very close relationship between logic and psychology. Mohanty (1982), in his book *Husserl and Frege* distinguishes between a weak and a strong psychologism.

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According to him, strong psychologism sees logic as a branch of psychology. Logical laws are psychological laws in the sense that they describe actual human thought processes, and they are empirical generalizations about mental events. On one hand, strong psychologism considers that the study of actual human thought processes constitutes both a necessary and a sufficient condition for the investigation into the foundations of logic (*ibid.* p. 20). On the other hand, weak psychologism restricts itself to say that essential theoretical foundations of logic lie in psychology; so that the necessity of logical laws like Non-Contradiction Principle is derived from the way in which our thinking works. In this sense, for weak psychologism, actual processes are a necessary but not a sufficient condition for the study of theoretical foundations of logic (*ibid.*). An example of the latter seems to be present in the work of the logician and mathematician George Boole. In his *An Investigation of the Laws of Thought* (1853), Boole refers to the supporting character of mental faculties and operations. He considers them as the foundation of the laws that constitute the science of logic (see [Boole 1853](#), pp. 12, 13, 44-45 and 62). Boole says that those mental operations can be accessed by observation, and their laws are “confirmed by the very testimony of the mind” (*ibid.* pp. 12-14).

Because of the empirical and subjective character of the so called “thought”, it can be said that logical laws are contingent, something that can exist in another way. This aspect has aroused different kinds of controversies because, from a certain point of view, it seems to be a weakness of psychologism that actually it does not give an account of the necessary and *a priori* character of logical laws¹. In addition, in the moment in which they attribute an empirical and a contingent character to logical laws, with or without intention, psychologists open the door to the possibility of a certain kind of relativism and a certain kind of pluralism in relation to those laws. From this perspective, different results in relation to heterogeneous individuals or human groups could be given by the previous research, and, in this way, different sets of logical laws could exist ([Alchourrón 1995](#), pp. 13-14).

This way of seeing the subject, in which, what is important for logic are thoughts, being understood as individual mental states and processes, involves the idea that also they are what is important in the field of language. In this context, language and thought are different things. Language would be just an instrument that serves to communicate those states and processes (which probably would remain hidden without language), and at the same time those states and processes would be responsible for the meaningfulness of language. This situation makes us suppose that logical psychologism implies a certain mentalism related to the notion of meaning, i.e., a position that states that the meaning of the expressions of language is given by subjective and private processes and representations². This allows us to think that any rejection of theories of this kind is accompanied by a rejection to psychologism. Thanks to this, antimentalism connected to meaning becomes a very important piece in the emergence of different ways of understanding the nature of logic. One can say that particularly, those different perspectives have interesting consequences for logical pluralism; that is, for the idea opposed to logical monism, that there is not a unique true logic, but rather many different true opposing logical systems. An approach to some alternatives will be presented in the following sections.

¹Apparently, someone who seriously questioned the necessary and a priori character of logical laws was John Stuart Mill, in his *System of Logic Ratiocinative and Inductive: Being a Connected View of the Principles of Evidence and the Methods of Scientific Investigation* (1843), (cf. [Alchourrón 1995](#)).

²A dark case for me is John Stuart Mill who, holding psychologism, defends a referential theory of meaning.

1.1 Frege's antipsychologism

Frege, as the psychologists above mentioned, talks about logic as the study of the laws of thought, but, he can be distinguished from them precisely because of the way he understands the word "thought". First, in his writing from 1892, the author introduces the concepts of *Sinn* (sense) and *Bedeutung* (reference) for pointing to two aspects of the meaning of the expressions of language. Proper names, concept-names and sentences have a kind of mode of presentation of the object (sense) and a designated object (referent). The antimentalistic character of this theory lies in the fact that Frege considers that the senses of expressions are not in any case mental representations. Senses for proper names, concept-names and sentences are objective entities. Second, Frege's philosophy holds the idea that logic is an investigation about what is true in an objective but not in a subjective way (Frege 1918-1919, pp. 289-290). The two theses described above converge in an anti-psychologist position that states: 1) what is interesting about logic is not the study of the subjectivity but of the senses of sentences (called *thought* by Frege), because they are the vehicle of truth; and 2) thoughts are not subjective representations of individuals but abstract objects whose basic characteristic is their objectivity. According to Frege, thoughts, like representations, are imperceptible to the senses, but differently to them, thoughts are accessible to different subjects; they are independent from the particular uses of language owing to the fact that one and the same thought can be expressed in different ways by different sentences of the same language or different sentences belonging to different languages. They also exist without being thought or taken as true by someone, which means that they pre-exist even before they are discovered; they are non temporal because they sustain their truth-value independently of the circumstances. All this leads Frege to consider thoughts as belonging to a "third realm" whose objects have an existence that is independent from the subjects.

There will be a more detailed discussion about Frege's conception of logic in the following section. For now, it is important to see that given the way in which things have been presented here, it looks like the only way to avoid psychologism is by choosing a certain Platonism. However, there are some philosophers who escape this dilemma, adopting a perspective of language that allows them not just to abandon psychologism in the context of logic, but Platonism as well. In the following, this paper examines and compares Carnap and Wittgenstein's points of view.

1.2 Carnap's logical conventionalism

At the beginning, Carnap's position coincides with Frege's in the fact that they both avoid psychologism and separate the field of logic from the field of psychology (see: Carnap 1934b, pp. 18-20). Nevertheless, Carnap takes distance from Frege in several aspects. In his period of *Logical Syntax of Language* (Carnap 1934a), Carnap defends the idea that language is a species of calculus. A language, then, is a system of rules formed by a finite set of symbols which constitute the vocabulary of that language, a finite set of *rules of formation* which indicate the kind of constructions that are acceptable as sentences within that language, and a finite set of *rules of transformation* which allows us to infer some expressions from others. The author compares his own conception with Hilbert's formalist proposal that supposes Mathematics is nothing else but only a system of symbols and rules in which every reference to meaning is excluded and that the only two important things are the order in which those symbols appear and the formal operations in which they have a role (see: Carnap 1934a).

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From this purely syntactic perspective, what is called “sense” or “content” of a sentence is neither the relationship of the sentence with a mental reality (thoughts or images in connection to the given sentence), nor, it could be said, with a Platonic reality (Fregean thoughts). Instead, what is called the content of a sentence is the set of sentences that are non-valid consequences of that sentence. A valid consequence is a sentence that is a consequence of premises of the null-class, i.e., the class that has no members. Primitive sentences (axioms) of *Principia Mathematica*, like P or $\text{not } P$, are considered valid consequences (*ibid.* pp. 26-28). According to Carnap, sentences of this kind are analytical, and they are ignored because they are consequences of any sentence (*ibid.* pp. 32-33). In this way, the content of a sentence is constituted by all those sentences that are consequences of that sentence, that are not primitive sentences of the system. For Carnap’s notion of language, logic is not more than an artificial system of signs and rules that excludes whatever reference to mental or Platonic meanings (Carnap 1934a, p. 27). Contrary to this conception of language as devoid of content, Frege’s position states that logical language, symbolic language, is not limited to represent an abstract logic in formulas, but its objective is “to express a content through written signs in a more precise and clear way than it is possible to do through words” (Frege 1882, p. 1 quoted in Goldfarb 2001, p. 27). Perhaps, in some aspects, the differences between Frege’s and Carnap’s conceptions of logic can be assimilated by the differences that Leibniz establishes between *lingua characteristica* and *calculus ratiocinator*. According to Goldfarb (Goldfarb 2001), one of those differences is the way in which they give an account of generality. While Fregean conception, called by Goldfarb *universalist conception*, gives an account of generality introducing a vocabulary that is not specific to any discipline, especially universal quantifiers and variables which allow us to express general laws, *schematic conception* does that by introducing “definitions that invoke all the interpretations of the given schemata” (Goldfarb 2001, p. 28). In other words, while the former achieves generality using a universal vocabulary, the latter does it by emptying expressions of content. Another important difference showed by Goldfarb appears in the context of application of logic. “In the schematic conception, logic is applied by passing from schemata to sentences that are their particular interpretations. In the ‘universalist conception’, applications are made by instantiating the quantified variables of a general law” (*ibid.*).

Frege’s and Carnap’s general positions about logic mark a series of differences between them. The first difference is related to the notion of truth and its consequences in relation to both; the task of logic and logical pluralism. For Frege, logic, as a system, consists of a series of propositions that are deduced from basic laws: logical laws. These laws are both prescriptive and descriptive. They are prescriptive because “rules for the asserting, thinking, judging, inferring follow from the laws of truth” (Frege 1918-1919, p. 289). And they are more general than physical laws, but as descriptive as those laws (*ibid.*) in the sense that they refer to an independent reality including not just objects but concepts, thoughts, and the Truth and the Falsity. Precisely, this descriptive character of logic is what leads to say that, to Frege, only one set of laws is correct, which means that, from this point of view only one logic is possible.

On the contrary, Carnap’s idea is that because of 1) in logic, the most important things are symbols and the rules of the game, the expressions themselves and their consequences, and not something that they could represent, and 2) the relation of consequence is dependent on linguistic structure, we can say that 3) “logical truths” and “mathematical truths” are not so by virtue of their relationship with something external to the language (truth by correspondence) but by virtue of the structure they belong to. In this sense, it can be

said that they are simply analytical sentences with which the user of the structure is committed (Goldfarb and Ricketts 1992, p. 64). On this point, truth is no longer understood as to be correspondence with something external to calculus but as deducibility within the calculus. And the task of logic is no longer the search for logical truths but a kind of task of classification of sentences that may be deduced within the calculus in some that are logical truths and some that are not (Chateaubriand 2001, p. 15). This way of seeing logical truths allows Carnap to state certain kind of conventionalism and pluralism that he reveals in his formulation of the Principle of Tolerance or Principle of Conventionality of Forms of Language (Carnap 1963, p. 102). In this regard, there could be diverse kinds of logic (each person could have his own), depending on the kind of vocabulary that has been chosen or the kind of formation and transformation rules that have been accepted.

Based on this, it is possible to say that the objective of logic passes from being the investigation about whether these ones or those ones are the laws of truth, and whether these rules constitute or not the right system, to be the task of construction and investigation of the formal calculus. The task is to determine the functioning of this or that element in the language that is being constructed, or determine the consequences of the sentences of the calculus if it is constructed in this specific way and not in the other (Goldfarb and Ricketts 1992, pp. 62-63). In Carnap's view, the choice of a logical system among others does not lie in its correctness in regard to mental or Platonic realities, "there is no right or wrong in the choice of a logic" (*ibid.*), but in merely practical reasons: "the practical reasons of preferring one or another form of language for a certain purpose" (Carnap 1963, p. 103). Thus, Carnap differs from Fregean descriptivism and logical monism, and in spite of the rejection of mentalism, he gets a little closer to psychologism in relation to the acceptance of a logical pluralism.

2 Grammar, Logical Laws and Logical Pluralism

We might find a perspective close to this in Wittgenstein's post-*Tractatus* work. Nevertheless, it might be said that in spite of his apparent proximity to Carnap, Wittgenstein gives the guidelines for a possible and interesting answer to Carnap's conventionalism. In some writings³, Wittgenstein develops a new way of doing philosophy narrowly linked to his notion of meaning as use. That new way is settled in certain kind of conceptual analysis (in some sense opposed to Russell's and *Tractatus's* logical analysis considering that here Wittgenstein abandons the idea that ordinary language must fit in an ideal language: *logical language*), and he proposes to look at the expressions in their actual and possible uses. This kind of analysis could be called *grammatical description*. It consists of presenting different perspectives of a concept, its uses in diverse contexts, or establishing similarities and differences between that concept and others in the framework of a calculus or a language game. Through grammatical description, he shows us the grammar of our language and allows us to see that the way in which it effectively functions does not necessarily suppose the existence of mental processes and states for making concepts meaningful (cf. Gómez 2006). However, this does not mean that there are no mental processes or states but that they are non distinctive because they do not *have* to exist for the concepts to be meaningful (BBB, pp. 72-73). According to Wittgenstein, those mental processes could be

³*Dictées de Wittgenstein à Waisman et pour Schlick* (Dictées), *Blue and Brown Books* (BBB 1933-1935), *Lectures on the Foundations of Mathematics* (LFM 1939) *Philosophical Investigations* (PI 1953), among others.

substituted by observable activities that involve observable objects which are supposed to serve the same function as mental objects. This substitution is possible not because of the way the real world functions, but (as shown by the grammatical description) because of the way language is actually or possibly used (*ibid.* pp. 29-30).

From this perspective, it can be said that the meaningfulness of language is independent of hidden processes, and actually, of any kind of extra-linguistic entity such as Fregean thoughts. For Wittgenstein, neither the meaning of the expressions, nor the fact of recognizing something as a word are either irreducibly psychological mechanisms or require strange processes of grasping. That is why he says that philosophy of logic talks about “the spatial and temporal phenomenon of language, not about some non-spatial and non-temporal phantasm” (PI, par. 108). With Wittgenstein’s perspective, both, meaningfulness and recognition of a word depend on the fact that words belong to a language. According to this, all that psychologists or Frege call *thought*, can be substituted by its expression. This means, it can be substituted by a statement that in terms of BBB “only has a meaning as a member of a system of a language, as an expression of a calculus” (BBB, p. 72), or in terms of PI, takes its sense from the way in which it is used within a language game (PI, par. 41).

2.1 Logic in the “calculus conception”

We can now, therefore, outline the way in which logic could be seen through the eyes of Wittgenstein after *Tractatus* (1921) as follows. First, in the *calculus conception* (Gerrard 1991)—this period goes from 1929 to mid 1930’s—logic can be understood, the same way as Carnap’s conception, as a certain kind of calculus or closed, self-contained system of rules. In other words, a set of signs, rules for the construction of expressions and rules to operate with them. These rules by themselves determine the meaning of expressions and constitute the last court of justification (Gerrard 1991, p. 126). In *Dictées, 1930*, Wittgenstein says the following when he refers to Russell’s logic:

(...) The word «to infer» refers, in our verbal use, to a completely determined calculus, namely, truth- function calculus. The rules of this calculus characterize what we have named as proposition, and the transitions within this calculus constitute what we have named inference. Now, Russellian axioms are given in such a way that transitions become logical inferences. This can be expressed in another way. Russellian axioms, we have said, are interspersed pieces in the schema that the rule of inference S designates, and with this rule they produce new rules of inference. Consequently, axioms are not in any way propositions of the language, because they just constitute the propositions of the language, they only say which propositions can be derived from others. Let’s put it in this way: axioms constitute the grammar of signs <not>, <or>, <if...then>, in a way in which their grammar corresponds to the grammar of those words just as we use them in our ordinary language. (*Dictées*, p. 99).

These rules -called *grammatical rules* by Wittgenstein (See also *Dictées*, pp. 93, 95, 98)- are in a certain way like Carnap’s syntactical rules. They are not propositions but just rules. This is why grammar is concerned in both cases. They are not a result of experience; they are not able to be true or false, and they have merely a normative character, and not a descriptive one (*ibid.* p. 132). Thus, they determine which expressions belong to the system, which are excluded, which place they occupy, which of them are synonymous, which

combinations are possible and which are not, in which cases it is possible to pass from one to another (to infer) and when it is not possible to do so, etc. This means that for Wittgenstein as for Carnap and different from Frege, the system of rules is not a system of truths resulting from a comparison with reality, but a set of “instructions for use” that can be compared and, in case of conflict, could be used alternatively (*ibid.* p. 118). Like Carnap’s idea of calculus, once these rules are fixed they make necessary the behavior of expressions and their relationships within the system. Also, in that idea, the system of rules is, say, independent and self-sufficient in the sense that rules are considered as the only criterion for the meaning (Gerrard 1991, p. 133), and, as it was suggested, the rules are arbitrary, that is, they do not obey to anything external, their fixing is independent of any reality-subjective or objective- external to the calculus. Rules are established by users. Therefore, talking about the specific case of rules of inference, the author claims:

(...) The truth is that the determination that one [proposition] follows from another is a grammatical rule that we fix. This means, the grammar of language defines which proposition follows from which other, and this is our arbitrary determination. (*Dictées*, p. 102).

This idea of arbitrariness accompanies the fact that for Wittgenstein and Carnap the system can function in many different ways and therefore, there may exist diverse systems depending on the signs and rules that are included. It is known that Carnap considers that syntax involves two basic notions: *sentence* and *immediate consequence*. On one side the definition of the expression “O sentence” (being “O” any language) is equivalent to the formation rules of that language. On the other side, the definition of the expression “immediate consequence in O” is equivalent to the set of transformation rules of system O (Carnap 1934a, pp. 25-26). These notions are both relative to one or another specific language, such that what can be considered a sentence or an immediate consequence depends on the formation and transformation rules in that specific language. For this reason, whatever change in the set of rules means a change in the application of the terms “sentence” and “immediate consequence”. But conversely, the entire syntactic structure of language is determined by what is considered sentence and immediate consequence in that language (Quine 1934, p. 76). For this reason, any change in a rule or a subset of rules means a change in the complete calculus, and it leads to talk about a different language or a different logic. Following this idea, Wittgenstein says:

Now, proceeding in this way, I provided a new rule for the signs $\langle \sim \rangle$, $\langle \supset \rangle$, $\langle \vee \rangle$, that does not correspond to the use of those signs as synonymous of the words $\langle \text{not} \rangle$, $\langle \text{if...then} \rangle$, $\langle \text{or} \rangle$. It would be like if I wrote the rule $P = \sim P$. Undoubtedly, I would have given a rule for the sign $\langle \sim \rangle$, but a different rule that that of the word $\langle \text{not} \rangle$; consequently, I would not be able to understand the $\langle \sim \rangle$ as a sign of negation anymore. But it is the same with the sign rule given above: in fact, we can establish it, but then we would destroy the essence of the propositional calculus, because neither the signs $\langle \sim \rangle$, $\langle \supset \rangle$ cannot be anymore understood as logical particles, and the signs p, q, r as propositions. (*Dictées*, p. 96).

In this paragraph it can be seen that Wittgenstein as Carnap considers that a change in the set of rules makes a change in all the calculus. When rules for the signs $\langle \sim \rangle$, $\langle \supset \rangle$, $\langle \vee \rangle$ are changed, the ideas about what is a logical particle and what is a proposition also change, and it occurs the same with the complete propositional calculus, because, in a

certain way, this specific calculus is defined by those notions. For this reason, a change in the rules of propositional calculus is not for Wittgenstein only a partial change. The other idea is that a change of calculus modifies the concept of proposition. In relation to this, Wittgenstein says:

It is usually believed that we have a concept of proposition that is independent from the calculus in a way in which that concept can be kept while the calculus is modified. Actually, the axiom is a part of a rule and the negation of the axiom now means something completely different: such proceeding that modifies the calculus totally and, as a result, modifies the concept of proposition. (*Dictées*, pp. 96-97).

Wittgenstein uses the term “proposition” in the place of the term “sentence” used by Carnap. However, it could be said that both have similar ideas. Wittgenstein considers that “propositional calculus is the essence of the proposition” (*ibid.* p. 188), this means that this set of rules is what constitutes the concept of proposition like “that that one can assert or deny, that that one can operate according to the rules of logic” (*ibid.* p. 187). Thus, if one accepts a sign $\sim P = P$, the concept of proposition would be affected because that rule would destroy the grammar of the primitive logical signs, i.e., the grammar of the words *< true >* and *< false >* that is what constitutes the essence of what is called proposition within that calculus (*ibid.* p. 96). In other words, one cannot operate with the sign $\sim P = P$ in the same way in which one operates with a proposition in the propositional calculus. One could deal with it in another calculus, different from propositional calculus. According to the author, a calculus that someday could become meaningful (*ibid.* p. 187). (This allusion to the future meaningfulness of the calculus, as if this were dependent of external causes, not from the calculus itself, turns out peculiar in this period of Wittgenstein’s work, in which two of the main characteristics of the system of rules are independency and self-sufficiency.)

All that has been said here makes evident that, from the perspective of the calculus conception, diverse calculi can exist. This allows us to make a link with another subject clearly present in some passages from *Dictées*. This refers to the idea that propositional calculus is not *the* calculus: it does not have any priority in relation to other calculi; the essence of logic is not crystallized in any way in it; there is no logical foundation that allows us to put it above the other calculi because any foundation, any justification is internal to the calculus itself, that is why every choice made between one calculus and another is completely arbitrary (*ibid.* pp. 188-189). This kind of logical pluralism is present in Wittgenstein’s later works as well. In LFM, for example, Wittgenstein considers the possibility of a calculus including contradictions. In PI, par. 554 he imagines the existence of a “more primitive” logic, something similar to Intuitionistic logic, in which only affirmative sentences can be denied, but in which the denial of negative sentences are nonsense. However, in this last stage, changes appear in some of the aspects mentioned here. These changes seem not to affect directly logical pluralism, but they do lead to a new perspective about it. Those changes are generated by the introduction of the notion of *language games* in the late 1930’s.

2.2 *Logic in the “language games conception”*

The notion of language game involves the idea that a language is or forms part of a form of life (PI, par. 19 and 23), i.e., a set of activities, practices, institutions. Contrary to the idea of calculus, the system of rules seen as a language game is no longer something isolated.

It is something anchored in a set of practices and it keeps a relation of continuity with other systems, now called “language games, thanks to the common base that those practices constitute (Gerrard 1991, pp. 138-139). It is necessary to make clear that this common base of practices or agreement in forms of life differs from the agreement in opinions. Better, agreement in opinions and agreement in definitions lie in the former (PI, par. 242). Making reference to the case of logical laws, in LFM XIX p. 183 Wittgenstein says:

This has often been said before. And it has often been put in the form of an assertion that the truths of logic are determined by a consensus of opinions. Is this what I am saying? No. There is no *opinion* at all; it’s not a question of *opinion*. They are determined by a consensus of action: a consensus of doing the same thing, reacting in the same way. There is a consensus but it is not a consensus of opinion. We all act the same way, walk the same way, count the same way. In counting we do not express opinions at all. There is no opinion that 25 follows 24 - nor intuition. We express opinions by means of counting.

According to the new position, rules are no longer those responsible for the meaning of expressions or the last court of justifications. Rules govern the use of signs, but now they are not independent of the context, they are determined by the context of human activities in which they are followed, by the set of practices in which those rules are involved. In other words, by the role they play in our lives (Gerrard 1991, p. 138; Lopes Azize 2004, pp. 270-271). From this perspective, one can say that the truths of logic, the logical laws are not empirical propositions, as supposed by psychologists. In fact, they are not considered propositions at all, they are not truth or false. They are instructions of use, rules, but rules that are determined by the set of practices and the forms of life in which they are involved. According to this, it can be said that the choice of a set of logical laws, the choice of a logical system, is not now something arbitrary, like Carnap seems to suppose, or decided by means of an agreement of opinions. Although, these elements were decisive in our choice, the final authority would be the role that certain expressions and the calculus as a whole would play in other language games and in general a certain form of doing things, a certain form of life. In LFM, Wittgenstein refers to arbitrariness, and there he alleges the following:

Now, in what sense are the rules of chess arbitrary? Suppose I said, “The colour of a pair of trousers is arbitrary”. As far as strength goes this might be; it certainly has no influence on that. But grass green trousers would hardly sell. You might say, “Another rule would have done just as well” For what? This suggests that there is nothing in the object of the game which determined this rule. Well- We don’t make up the rules of these games. Chess and similar games we have *inherited*. (LFM XV p. 143).

Certainly, nothing exists, within the system, to prevent us from changing the rules we desire or creating the system we desire, but the fact that those new rules are fixed, the fact that the new system is imposed, or even that it is considered useful depends on a form of life (sometimes Wittgenstein talks about “imagery”) inherited by us or a creation of a new one. This form of life is what makes the use of a rule something *natural* or something *unnatural* (LFM XIX pp. 183 and 186-187). In a try to show the sense in which he uses the concepts natural and unnatural, Wittgenstein presents the following example:

If I give a man a table of colour samples with the name “sea-green” under one of them, and then say “bring me a sea-green book”, it would be highly unnatural if, instead of

looking round for a book the same colour as the sample, he were to look round for the complementary colour. But he might do so. (*Ibid.* XX p. 193).

Looking for the sea-green complementary color book seems unnatural since -by the training we have received or by those behaviors we have been accustomed to-, say, we feel it strange at first sight. An interesting case related to this is precisely that that involves the acceptance of contradictions within a system. Perhaps it can be said, without betraying Wittgenstein, that one of the reasons why we normally do not accept a calculus with contradictions is that, if we include them, the game that would be played would be different from the game that we are accustomed to play. On one hand, the rules for the use of certain signs like negation and conjunction would change and we would be giving to words for negation and conjunction a wrong meaning. On the other hand, "If we allow contradictions in such a way that we accept that anything follows, then we would no longer get a calculus, or we'd get a useless thing resembling a calculus" (*ibid.* XXV p. 243).

Wittgenstein has a different way to think about both topics. On one side, for him, the meaning of an expression does not go beyond its use within a certain language game, practice or a form of life. Therefore, if we introduce a new rule in a certain language game, that new rule, that new use of the expression is the meaning of that expression. There is no meaning independently of the use. In this sense, there is not anything like a wrong meaning. There are mostly unnatural uses (LFM XIX p. 183) or useless games (*ibid.* XXI, p. 207). Precisely, in LFM XX p. 191, the author talks about the meaning of the negation: "What use of a word characterizes that word as being a negation? Isn't it the *use* that makes it a negation? It's not a question of our first having negation, and then asking what logical laws must hold of it in order for us to be able to use it in a certain way. The point is that using it in a certain way is what we mean by negating with it" (see also PI, par. 547-561). On the other side, Wittgenstein seems to consider that there are no logical reasons for not accepting contradictions or saying that they do not work. Better, when we talk about logic, we think in a certain way of calculating and in a certain way of thinking that is what we, in fact, practice. In other words, we think of the language technique that we all know well, which normally excludes contradictions or just includes isolated cases that are generally devoid of meaning. For this reason, a calculus including contradictions seems unnatural and we are tempted to call it *illogical* (*ibid.* XVIII p. 179 and XXII 213). Nevertheless, reaffirming, in a certain way, a kind of relativism and its consequent pluralism, Wittgenstein emphasizes that although we can say that this is the case *for us*, i.e., "this is just one logic, and in others you may have as many contradictions as you like" (*ibid.* XXII p. 213).

In Wittgenstein's perspective, the discomfort that we feel at first sight, the sensation we have that something "jams" may change. The imagery can be changed and in this context the new rules can become natural. And the subject involved here is the will:

We most naturally compare a contradiction to something which jams. I would say that anything which we give and conceive to be an explanation of why a contradiction does not work is always just another way of saying that we do not want it to work (...) one needs to change one's imagery in the case of contradictions. One can change one's imagery in such a way that "p and not-p" sounds entirely natural, as when we say, "the negative doesn't add anything". This is most important. We shall constantly get into positions where it is necessary to have a new imagery which will make an absurd thing sound entirely natural. (*Ibid.* XIX p. 187).

So, while in the calculus period a change in the rules meant just a change in the whole calculus, now it exceeds the frontiers of the calculus and supposes a change in a certain way to do the things, in a series of practices, in a form of life. And at the same time, it demands a change of will, maybe not simply in the will of only one person but in a complete linguistic community, because of the public character that language has for Wittgenstein. Maybe going in the same direction in relation to the role that forms of life and willingness have in the evaluation that we make of certain rules and certain systems, the author says in PI: “Civil state of contradiction, or its state in the civil life: there is the philosophical problem” (PI, par. 125). When a new calculus like that imagined by Wittgenstein is appraised, the point is not simply the change of calculus in an arbitrary way or as a result of an agreement of opinions. Contrary to our suppositions, the question is to see contradictions have a place in our technique of language, in our way of thinking and acting. Despite the fact that some contradictions constitute useless language games, some others have a role in diverse language games, in certain practices and in general in our forms of life. Wittgenstein gives some examples of contradictions that have a function. He talks about a contradictory order that we can use to make someone astonished or paralyzed, or make him act with hesitation. Also, he talks about some cases in which an order expressed in the second place is used to cancel an order expressed in the first one. Perhaps also one could include in this list: contradictions used to make jokes or involved in the description of certain emotional states; or contradictions that serve to identify a murderer, or used in historical research for establishing historical facts; or contradictions involved in some images of christianity or contradictions that appear in Zen koans and in the comments of some Zen Masters.

In relation to this, in R. G. Collingwood’s *The Idea of History* (1946), one can find both an analogy between the role that contradictions can play in the context of crime investigations and in the context of historical research, and indications about the way in which it is possible to work with them in those realms; and Douglas R. Hofstadter, in *Gödel Escher, Bach: An Eternal Golden Braid* (1979), considers the role that some paradoxes may play in the context of Zen Buddhism. All these could be examples of contradictions that have a function, in the sense that, in some cases they seem to be constitutive of certain speeches and acts, and in other cases they lead us to make the inferences that we make. For example, if, in the case of the joke, the contradiction were eliminated, probably there would be no joke; without the contradiction between the humanity and the divinity of Jesus Christ, there would be no Christianity; if we would remove the contradictions when describing some feelings or emotions, maybe such a description would be incomplete. And, in the case of the crime, it can be said that it is the contradiction what starts the inferential chain: the contradiction makes us think that X is lying, and that thinking allows us to infer that X is guilty or, as in the story narrated by Collingwood, that X is harboring the real murderer.

It is important to take into account the current use of contradictions because it is precisely its support in a form of life, its status in the civil life, and its relationship with other language games (like mathematics, physics, linguistics, history and computer science among others) what makes the calculus involving contradictions or whatever calculus be useful or be accepted in common agreement by a community. In the process of changing the way of *seeing* things, the Wittgensteinian philosopher has a very important place, due to the fact that it is by means of the specific type of analysis he makes that it is possible to see the multiple uses that an expression of our language can have. Throughout the grammatical description, the philosopher shows us, for example, the role that certain contradictions can accomplish within a certain language game, the relationships that this language game can

have with others, the role or roles that these contradictions may have in our form of life and the different ways in which it is possible to deal with them. Remembering or imagining new situations of use (PI, par. 122 and 127), the philosopher, in a therapeutic attitude, tries to persuade us. He tries to change our way of seeing the subject of contradictions and our initial rejection of them. As a result of this, we will be able to see how useful a calculus involving contradictions can be, and besides, we will have a support to say that one calculus is better than another, obviously, relative to certain language game, set of activities or form of life.

In his work, Gerrard makes us notice the existing differences between the calculus conception and the language game conception related to the problem of comparison between calculi. He remarks that while in the calculus period whatever change in rules constitutes a change of calculus, and it is not possible to compare one to another, in the language game period the resulting calculus does not have to be considered a new and alien calculus. In virtue of the common ground furnished by the common ways of doing things and the activities shared in a determinate form of life, the new calculus can be compared with the old calculus and, in some cases, the new one can be considered an extension of the old one. Gerrard relates this situation to the fact that, while in the old conception we can just talk about each particular calculus, in the new conception, we can refer to calculus as a whole. He supports his statement with a reference to PI, par. 67 where Wittgenstein talks about the *family resemblances* that different language games can have and the kind of family that those language games can constitute (Gerrard 1991, pp. 135-136).

Gerrard's reference to PI, par. 67 is appropriate to show the difference between the calculus conception and the language game conception in the possibility that exists in the latter to make comparisons among diverse calculi, and to establish relations or family resemblances among them until we can include all of them in a family: the family of calculi. However, one can doubt the step that goes from here to the statement that consistency is a necessary characteristic for all calculi: "But when considered from the language-game conception consistency is *not* an optional characteristic of calculi, nor it is a superstition" (*ibid.* par. 137). Gerrard points out that for Wittgenstein every calculus, like every game, besides the rules, has a *point*, and he also states that inconsistency, i.e., the presence of contradictions, destroys the point of the calculus. According to Gerrard, the reason is that the main characteristic of calculus is that it does not permit the deduction of false conclusions from true premises. If the calculus were inconsistent it would be possible to make this kind of deductions and, in fact, any kind of statement could be deduced from it. Thus, everything we were expecting to do with the calculus, everything we expected from it fails. There are some elements that allow us to answer Gerrard in this aspect. First it is what in PI Wittgenstein says:

Why do we call something a "number"? Well, perhaps because it has -direct- relationship with several things that have hitherto been called number; and this can be said to give it an indirect relationship to other things we call the same name. And we extend our concept of number as in spinning a thread we twist fibre on fibre. And the strength of the thread does not reside in the fact that some one fibre runs through its whole length, but in the overlapping of many fibres. (PI, par. 67).

This passage points to a very important difference between Frege and Wittgenstein in relation to the way of understanding the reference of concepts. This difference relates us to the question about identity criteria for a calculus. Summarizing, for Frege the function of a concept is intentionally defined specifying the common features of the objects that can fall

under that concept. Based on the quoted passage one can say that, for Wittgenstein, the extension of a concept, like “number” or, let’s say, “calculus”, is not defined from a set of properties common to all numbers or all calculi, but, better, from a certain family resemblance existing among certain things, some of which have been called, until now, number or calculus. Consequently, the difference between Frege and Wittgenstein would be given by that Fregean supposition of common features, which in Wittgenstein becomes something less definite: the family resemblances. Then it can be said that, since there is not a common base to everything that constitutes a calculus, there is no reason to suppose that consistency is necessarily what permits us to decide whether it is or is not a calculus.

Second, in LFM XXI p. 205, as in PI, par. 142, 564 and 567 Wittgenstein remarks how important is the *point* in the use of language or whatever game, but also he seems to recognize the relativity of the point if it is understood as that that can be considered essential in a game:

A use of language has normally what we might call a point. This is immensely important. Although it’s true this is a matter of degree and we can’t say just where it ends. (...) We have, apart from any table of rules, an idea of the point of a game. -But what is regarded by one person as essential may be regarded by another as inessential; and it isn’t always a clear issue. (LFM XXI p. 205).

Third, the problem with inconsistency seems to be that it leads to triviality, i.e., whatever conclusions can be drawn from propositions of the calculus. Nevertheless, everything seems to indicate that Wittgenstein considers possible alternatives to this situation, making a separation between consistency and triviality, as it is made by paraconsistent logics but in a different way. Marcos 2001, p. 9 shows that Wittgenstein considers the possibility of controlling the effects of the contradictions and avoiding triviality introducing rules *ad hoc* or a general rule like “this is not use - and we wont draw any conclusions from it” (LFM XXI p. 209). However, Marcos notices that the solution proposed by Wittgenstein, namely, to take the healthy part of calculus available before the contradiction had appeared, is not very useful because the contradiction can be generated in any other part of the calculus:

Now, in fact, there is a great part of Frege’s calculus that is independent from contradictory premises, but the fact is also that the contradiction can still be generated within the calculus with no great effort. (Marcos 2001, p. 11).⁴

Finally, when Wittgenstein talks about the relationship between inconsistency and triviality he does it by showing how it is that we think and why we reject contradictions. But, as noted above, he also states that when we think about logic, we think about our own way of calculating, and that the rejection of contradictions is proper to a logic, our logic; the logic that we know, but there might be other logics involving the number of contradictions that is desired. In addition to this, perhaps the most important thing, we see how Wittgenstein insists on the idea that we do not have to see the things all the time in the same way. There is always the possibility that, by an effort of the will and with the help of the therapist philosopher, what we have seen before as a duck we can now see as a rabbit and vice versa.

The interpretation given above raises the question about the limits of pluralism. Here, it has been said that just an effort of will and the work of the philosopher are enough to see the role that certain kinds of expressions have in our language and to accept a system

⁴The translation of the quotes from Marcos 2001 is mine.

of rules which includes them. Does it mean that every calculus that can be thought of is acceptable? Gerrard's interpretation makes us think clearly that the introduction of rules has limits. For example, we have seen that for him contradictions cannot be accepted because the essence of calculus would be lost. However, the cost of this is that we would have to accept something that Wittgenstein might not accept: the definition of concepts by means of characteristics common to all objects that fall under those concepts. Another way to answer this question is through the idea of applicability; to be exact, by means of recognizing the uselessness of certain expressions or certain linguistic techniques. Something related to this can be found in the following passage in which Wittgenstein refers to the supposed arbitrariness of grammar. The general idea is that the decision about which expressions can or can not be considered as propositions is a matter of grammar. In relation to the supposition that grammar is too arbitrary for that effect, he says:

So does it depend wholly on our grammar what will be called (logically) possible and what not, -i.e. what that grammar permits? -But surely that is arbitrary! -Is it arbitrary?- It is not every sentence-like formation that we know how to do something with, not every technique has an application in our life; and when we are tempted in philosophy to count some quiet useless thing as a proposition, that is often because we have not considered its application sufficiently. (PI, par. 520).

In the context of the language game conception, the meaning of expressions does not depend just on a set of rules but also on the use of those rules in a certain language game or, in general, in a certain form of life. There are expressions with an use, i.e., techniques with an application within a language game or within a form of life, and there are others completely useless. Precisely, this aspect would seem to be the filter for the selection in a range of different calculi. In the same way in which the use of certain expressions, the application of certain techniques would support the introduction of a new calculus -remember the calculus of contradictions whose acceptance would be supported by the fact that in our language there are contradictions that are useful within a language game or within different language games- their lack of applicability could lead us to reject a calculus that includes them. However this answer does not seem so emphatic in favor of a limitation of pluralism, because, as Marcos says in his article, "it is clear that with just a little of imagination and willingness we always could conceive one application" (Marcos 2001, p. 14). In other words, we come back to the point of the "everything is acceptable".

This debate between the idea that "everything is acceptable" and "limits exist" can be seen as follows: we could say that effectively Wittgenstein considers that all expressions could have a use or all techniques could have an application and that, therefore, many kinds of calculi are possible, being just a matter of imagination and willingness, or simply memory, to find that use or application. However, at the same time, he is conscious of the limits of pluralism in the behaviours and habits that we inherited, in the way in which we currently see things. He is conscious that the force of tradition and the training we have received for generations is so strong, that it requires a great effort of the will to change the imagery necessary. This effort has to be greater if we take into account the fact that, given the public character of language, it is not only one person but the linguistic community as a whole that has to change its mind. The value of a view like this consists of being half way between Frege's logical monism and Carnap's pure conventionalism. Wittgenstein recognizes the force that both heritage and education give to propositional calculus and the limits that they impose over the introduction of new calculi, but he opens the door for logical pluralism

as well, considering as the criteria for the acceptance of new calculi the use or applicability of certain expressions or language techniques.

It can be said that, in the language game period, Wittgenstein considers that the introduction of a new calculus cannot be reduced to a simple change of rules. It demands an awareness of the way in which words are used and a change in the way of *seeing* things. In doing so, he not only calls for tolerance. He also gives philosophical bases for the acceptance of logics different from propositional calculus, by means of : a) the introduction of a notion of meaning as use or applicability that recognizes the meaningfulness to linguistic expressions, independently from the existence of extra-linguistic entities that could correspond to them. Meaning, understood as use, grants meaningfulness to new expressions, allows us to introduce new meanings to already existing expressions, and allows us to recognize meaningfulness to expressions that, in another way, would be treated like non senses (that is the case of some uses of negation and conjunction); and b) the introduction of a certain kind of analysis that allows us to be conscious about the different uses or applications of concepts, and of the way in which language indeed behaves or could behave without the intermediation of ideal structures that serve as models to follow.

The intention in this paper has been to highlight the role that could be played by Wittgenstein, in his post-*tractatus* work, in discussions about the nature of logic and logical pluralism, presenting the Wittgensteinian perspective as a kind of antipsychologism different from both Frege's and Carnap's. It was specially important to try to show that the non mentalistic and non Platonistic conception of language presented by Wittgenstein in his last works, and the notions of use, language games and forms of life included in it, make possible for him to consider logical laws as being normative and not descriptive, and allow him to defend a logical pluralism at the same time that he answers to the kind of conventionalism proposed by Carnap. As a part of this development, this work has presented the specific case of contradictions, and the possibility of existing a calculus that includes them, as an example of the way in which logical pluralism could be understood in the context of Wittgenstein's late philosophy.

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